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Analysis of Salt Water Intrusion in Coastal Aquifers

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ABSTRACT. The position of salt water-freshwater interface in coastal aquifers is analyzed. Governing partial differential equations are formulated by combining Darcy's Law and continuity equation and by using Ghyben-Herzberg Principle and Dupuit assumptions. For the solution of the governing equations Finite Difference Method is used. A computer program based on two-dimensional steady-state mathematical model is utilized. As sufficient and reliable field data are usually not available, the program is tested using two simplified flows for which analytical solutions are available or obtainable, namely, a one-dimensional flow in a strip aquifer with two different boundary conditions and an axi-symmetric radial flow in a circular aquifer. The results revealed that there is a good agreement between analytical and numerical solutions. Finally, after sensitivity analysis and calibration, the model is applied to the lower valley of Wadi Hali on the coastal plain of Saudi Arabia. The application is intended to be an example of a management scheme. The results are discussed and some conclusions are drawn. Limitations of the model arise from the assumptions on which it is based. Therefore, the application of the model is limited to steady, two-dimensional flows in homogeneous and isotropic aquifers in coastal regions.

Introduction

In coastal regions, aquifers generally constitute a substantial part of water supply. With increasing urbanization, agricultural, industrial and other activities, the need to develop the groundwater aquifer increases. However, inadequate data and often hasty decisions in the development process of the aquifer results in surpassing its safe yield. This in return results in sea water intrusion which is one of the major problems with regard to water quality in such areas.

In coastal areas, if the aquifer is in hydraulic contact with the sea, the lighter freshwater overlies the heavier salt water. Under normal conditions, there exists an equilibrium between these two liquids which differ in their densities. The movement of the freshwater, with a seaward gradient of water table, towards the sea acts as a barrier to the intrusion of the saline water into the aquifer (Henry 1959). In general the hydraulic gradient is modified either by natural hydrological events such as inadequate recharge by infiltration and excessive evapotranspiration or by man's intervention such as overdrafting by excessive pumping and artificial recharge. If the normal seaward gradient of the groundwater table is flattened or reversed to landward gradient, the seawater intrusion occurs.

As a result of hydrodynamic dispersion, generally, a transition zone develops between fresh and saline waters. However, this zone is quite narrow as compared to other dimensions of the aquifer. As a consequence, for most practical purposes the problem is usually approached by assuming a sharp interface. When this interface advances and reaches pumping wells, the quality of the freshwater in the vicinity is deteriorated and becomes unsuitable for the purpose of water supply. The contamination of the freshwater by sea water intrusion manifests itself primarily through the change in the chloride ion content. The chloride ion of freshwater is generally less than 100 parts per million (ppm). As it increases beyond 400 ppm, the utility of the water decreases. Sea water commonly has 19000 ppm of chloride. Therefore once the sea water infiltrates a well, the quality of the water pumped will deteriorate rapidly, often within a short period (Keith 1977).

In the light of the above discussion, it is apparent that the prediction of the position and movement of the salt water-freshwater interface which results from natural and/ or man-induced events mentioned above is invaluable to the proper management and protection of the coastal aquifers.

Theoretical Background

1. Literature Review

The earliest work related to sea water intrusion problems was undertaken by Badon-Ghyben (1888) and Herzberg (1901). Their work has been known as Ghyben-Herzberg Principle and it marks the beginning of quantitative analysis to locate the salt water-freshwater interface in coastal aquifers. Since then, numerous investigations have been carried out and therefore a considerable amount of references are available in the literature. For a comprehensive literature review on quantitative analysis of salt water-freshwater relationships in groundwater systems, the reader is referred to, for example, Reilly and Goodman (1985).

From the methodological point of view, techniques used so far have been experimental, analytical and numerical. Each group has advantages and disadvantages.

Experimental methods usually consist of physical models such as the Hele-Shaw apparatus or the Sand-Box [Bear and Dagan 1964a, Dagan and Bear 1968, Collins and Gelhar 1971]. Their use is primarily directed to the understanding of the mechanism of the salt water intrusion and to the verification of the analytical and numerical solutions.

Numerous analytical studies have been carried out to study the freshwater-salt water interface [Bear and Dagan 1964a, Glover 1959, Henry 1959, Henry 1964, Bear and Dagan 1964b, Rumer and Shiau 1968, Hantush 1968, Mualem and Bear 1974, Strack 1976, Kishi and Fukuo 1977, Van der Veer 1977, Isaacs and Hunt 1986]. Almost all of these studies are based on simplifying assumptions, which, in practical problems, are not always justifiable. Hence, numerical models are the only way to handle such cases.

Many investigators, consequently, have worked on the sea water intrusion using sharp interface approach and employing numerical techniques. In these works, the problem is treated in either horizontal or vertical plane. Some authors used finite dif ference method [Shamir and Dagan 1971, Fetter 1972, Anderson 1976, Ledoux *et al.* 1990]. Some others employed finite element method [Sa da Costa and Wilson 1979, Layla 1980, Sa da Costa 1981, Contractor 1981, Contractor 1983]. Boundary Integral Method has also been used to solve the same problem [Taigbenu *et al.* 1984, Liu and Liggett 1978].

Several authors addressed the problem of salt water intrusion using miscible flow approach that takes into account the effect of the diffusion-dispersion phenomenon. The governing differential equations are generally solved by numerical methods [Pinder and Cooper 1970, Pinder and Page 1976, Kono 1974, Lee and Cheng 1974, Segol *et al.* 1975, Andersen *et al.* 1988].

Numerical methods have the general advantages of accuracy and flexibility, but are complicated to apply [Isaacs and Hunt 1986]. Further, if they are sophisticated, although theoretically sound and pleasing, they become too cumbersome for routine use by the practicing professionals. In addition to these, the models mentioned in the literature are mostly unavailable or very poorly documented [Sa da Costa and Wilson 1979].

In the light of the foregoing discussion, it becomes apparent that there is a need for a simplified yet accurate simulation model. This model should make a compromise between oversophistication and oversimplification.

2. *Governing Equations*

In coastal aquifers where the thickness of freshwater is usually thin compared with the lateral extent, and the slope of water table is small, the groundwater flow in the aquifer generally assumed to be horizontal under normal conditions. In such a case the equipotential surfaces are vertical and the velocity is uniform over the depth of the flow. Groundwater problems in such aquifers are almost always approached using these approximations (Fetter 1980). They are referred to as Dupuit-Forcheimer assumptions.

In addition to the above, in this study, the following simplifying assumptions are used in the mathematical formulation: the aquifer is both homogeneous and isotropic; fresh and salt waters are immiscible and a sharp interfce exists between them and there is no flow across this interface; fresh and salt waters are slightly compressible and their viscosities are constant; sea water is stagnant and tidal action is negligible; vertical flow is only considered for accretion.

The mathematical model describing the position of the groundwater table and the depth of the salt water interface, both with respect to the mean sea level, may be derived using simultaneously: a) principle of conservation of mass, b) Darcy's law, c) Ghyben-Herzberg principle.

Typical cross-section of a coastal unconfined aquifer is depicted in Fig. 1. The governing partial differential equation which describes the unsteady position of the water table, h, for this flow system is:

$$K\,(1+\delta)\,[\frac{\partial}{\partial x}(h\frac{\partial h}{\partial x}) + K\,(1+\delta)\,[\frac{\partial}{\partial y}(h\frac{\partial h}{\partial y})] + W = S\,(1+\delta)\frac{\partial h}{\partial t} \quad (1)$$

with $\delta = \gamma_f/(\gamma_s - \gamma_f)$. The symbols used are as follows: K-hydraulic conductivity [L/T], h-water table elevation [L], W-recharge rate [L/T], S-specific yield, dimensionless, x-space coordinate [L], y-space coordinate [L], t-time [T], γ-specific weight of water [M/T^{2}L^2]. The subscripts f and s refer to fresh and salt waters, respectively.

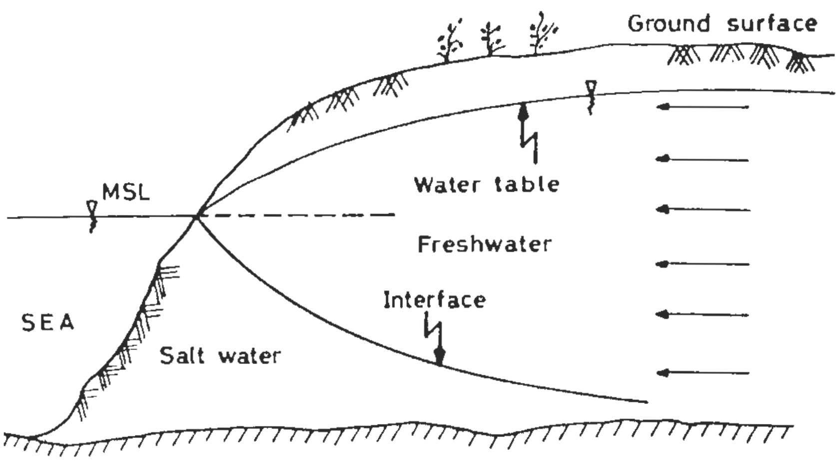

FIG. 1. Typical cross-section of a coastal unconfined aquifer.

The depth of interface with respect to mean sea level is then given by Ghyben-Herzberg relationship.

$$\eta = \delta h \quad (2)$$

where η-depth of interface with respect to mean sea level [L].

Details for the derivation of the above equations may be found elsewhere (Onder 1989). Slightly different derivation is given by Bear (1979).

When the flow is steady, the flow parameters remain constant with respect to time, *i.e.*, the right hand side of Eq. 1 becomes zero, and it takes the following form:

$$K\,(1+\delta)\,[\frac{\partial}{\partial x}\,(h\,\frac{\partial h}{\partial x}\,)\,+\,K\,(1+\delta)\,[\frac{\partial}{\partial y}\,(h\,\frac{\partial h}{\partial y}\,)]\,+\,W=0 \tag{3}$$

When it is further manipulated, it becomes

$$\frac{\partial^2 h^2}{\partial x^2}+\frac{\partial^2 h^2}{\partial y^2}=\frac{-\,2W}{K\,(1+\delta)} \tag{4}$$

When h^2 in Eq. 4 is replaced by a new variable v it would easily be recognized as the Poisson Equation.

For a complete mathematical description, these equations must be accompanied with a set of appropriate boundary and initial conditions. They are dictated by the particular problem under consideration.

The solution of the resulting equations by using an appropriate technique gives the elevation of the water table with respect to mean sea level. The depth of the interface is then easily obtained using Ghyben-Herzberg principle (Eq. 2).

3. Numerical Model

For the solution of mathematical model, a well known numerical solution technique, namely finite difference method, is adopted. Using central difference approximation, Eq. 4, at any nodal point $(i,\ j)$ may be approximated as:

$$\frac{v_{i+1,j}+v_{i,j+1}\,v_{i-1,j}+v_{i,j-1}-4v_{i,j}}{(\Delta x^2)}=-\,\frac{2W_{i,j}}{K\,(1+\delta)} \tag{5}$$

where $v_{i,j}=h^2_{i,j}$. The remaining symbols are as follows: Δx is space increment in x direction, K is hydraulic conductivity, W is net recharge rate, and δ is a constant. The increment in y direction is taken to be equal to Δx.

Applying Eq. 5 to all nodal points will give a system of approximating algebraic equations. This set of n equations in n unknowns can be written in indicial notation as:

$$a_{i,j}\,v_i\,=\,b_i \tag{6}$$

where $v_{i,j}$, i and $j=1,2,\cdots,n$ are unknown parameters; b_i, $i=1,2,\cdots,n$ are known values; a_i, $i=1,2,\cdots,m$ are known coefficients.

Equation 6 can also be rewritten in matrix form, using capital letters to denote each array, brackets to denote a square matrix, and braces to denote a column matrix.

$$[A]\{V\} = \{B\} \tag{7}$$

In general, linear algebraic equations are solved by iteration or direct methods. In this work, the set of linear algebraic equations given by Eq. 7 is solved by Gauss Elimination Method, which is a feasible direct method, when matrix equation contains less than 1000 unknowns.

The listing of computer program used in the solution of Eq. 7 and the details of it may be found in Onder (1989).

Verification of Numerical Model with Analytical Solutions

Due to lack of adequate and reliable field data, comparison of numerical results with actual field observations may not be possible. An alternative way of verification is to test the numerical model against known solutions to the partial differential equation. This is the case in the present work.

In this context, to demonstrate the accuracy of the solution technique employed in the numerical model, three analytical solutions are used, two of which are for one-dimensional steady flow in an infinitely long strip aquifer and the third one for axi-symmetric radial flow in a circular island.

Steady state numerical model developed in this work have been applied to the problems mentioned above. It has been observed that there is a very good agreement between analytical and numerical solutions. The results are presented and compared in graphical form.

Cross-sectional view of the flow system in an unconfined coastal strip aquifer between sea and a freshwater lake along the width is depicted in Fig. 2. The width of the aquifer in x-direction is L, and the aquifer has an infinite length in y-direction. In the absence of point source (recharge well) or sink (discharge well), the flow is practically one-dimensional in x-direction. The computed values, by numerical method, of the water table elevations and the depth of interface at nodal points lying on the x-axis are plotted in Fig. 3. On the same figure corresponding analytical results are also shown. It is seen that the agreement between numerical and analytical results are almost perfect.

A sketch of a vertical cross section of an unconfined coastal strip aquifer bounded on one side by the sea and on the other by an impervious formation is depicted in Fig. 4. Let the origin $x = 0$ be located at the intersection of the interface with the no-flow boundary of the aquifer. Seaward freshwater flow at this section is zero. The values of the water table elevations and the depth of interface at nodal points laying on the x-axis are plotted in Fig. 5. On the same figure corresponding analytical results are also shown. It is seen that there is a very good agreement between numerical and analytical results.

The vertical cross-sectional view of an unconfined aquifer beneath a circular island along the diameter is given in Fig. 6. The computed values of the water table eleva-

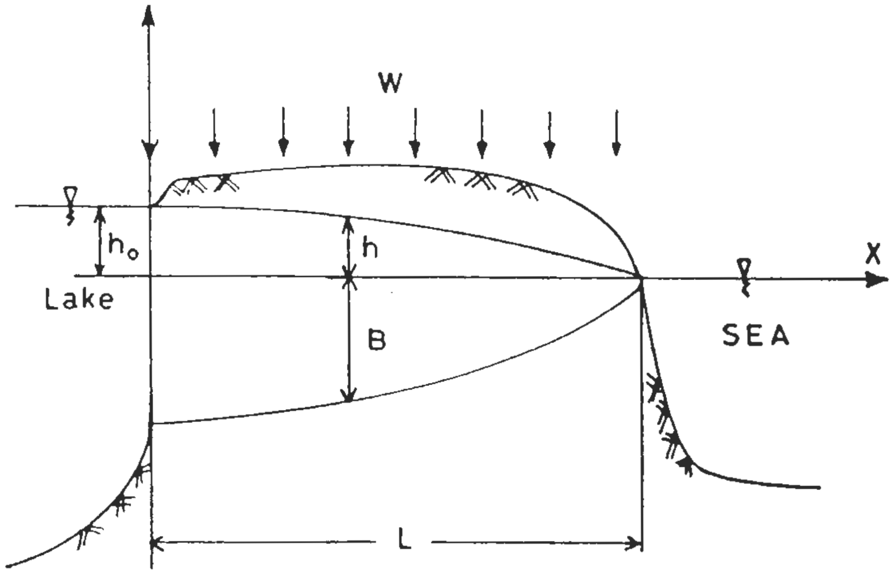

FIG. 2. One-dimensional steady flow in an unconfined strip aquifer between sea and a freshwater lake.

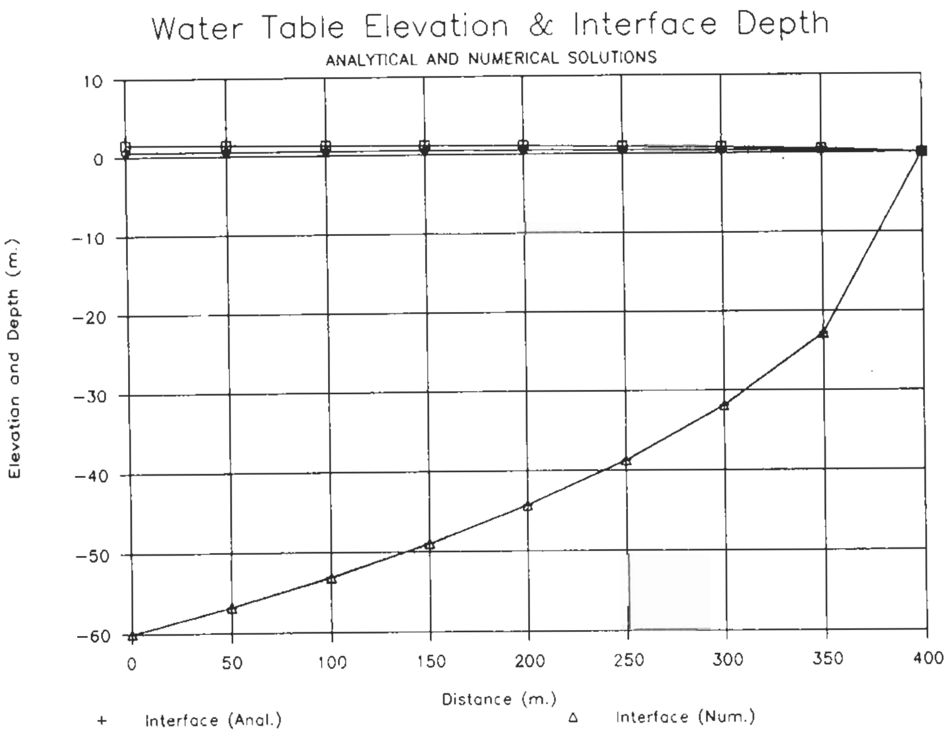

FIG. 3. Elevation of the water table and the depth of the interface along x-axis (Comparison of analytical and numerical solutions for Case 1).

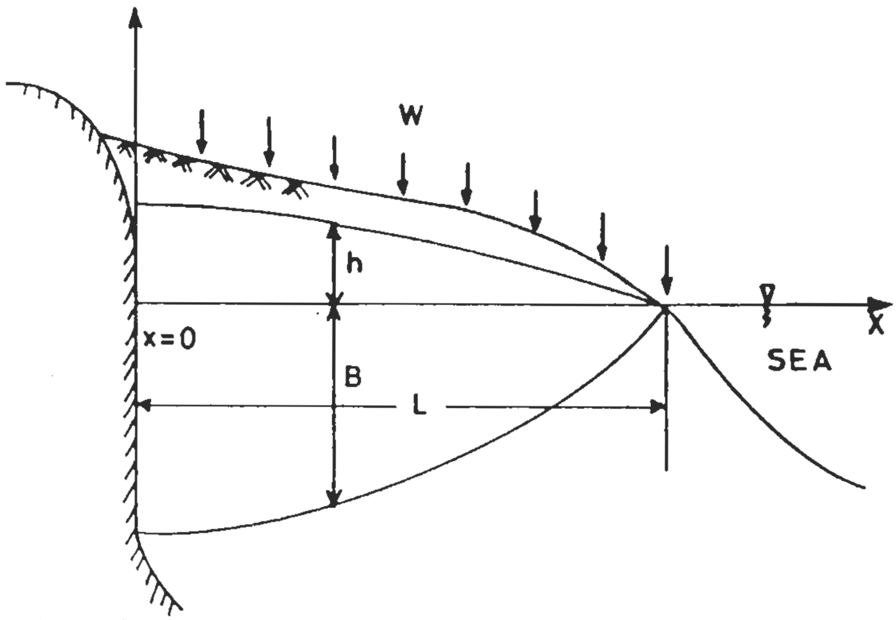

FIG. 4. One-dimensional steady flow in an unconfined strip aquifer between the sea and an impervious boundary.

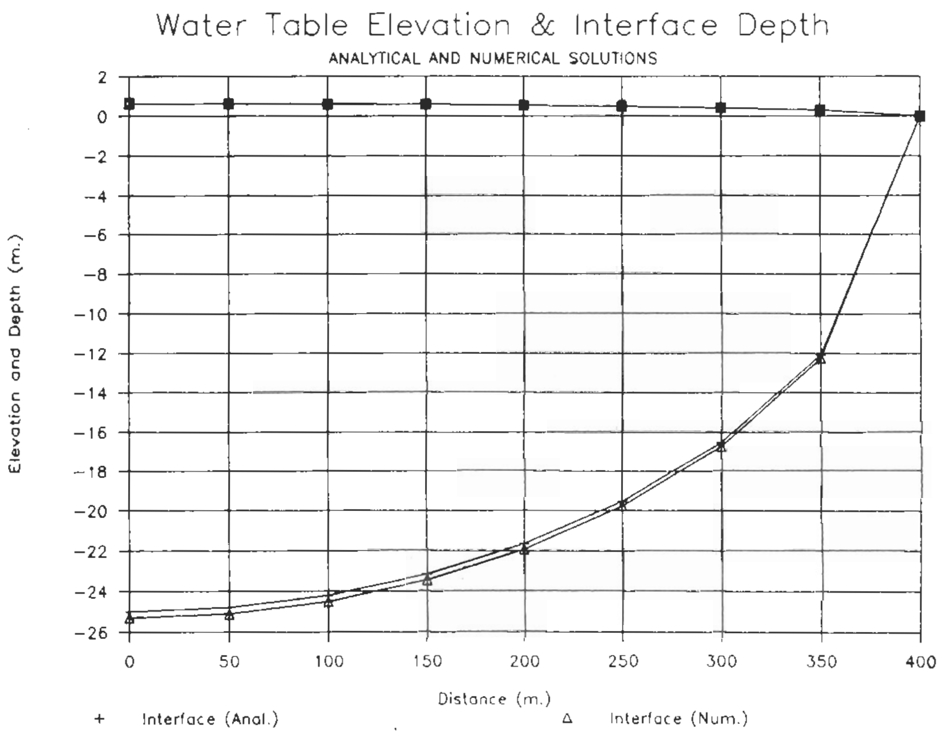

FIG. 5. Elevation of the water table and the depth of the interface along x-axis (Comparison of analytical and numerical solutions for Case 2).

tions and the depth of interface at nodal points laying on the diameter are plotted in Fig. 7. On the same graph corresponding analytical results are also shown. It is seen that there is a very good agreement between numerical and analytical results.

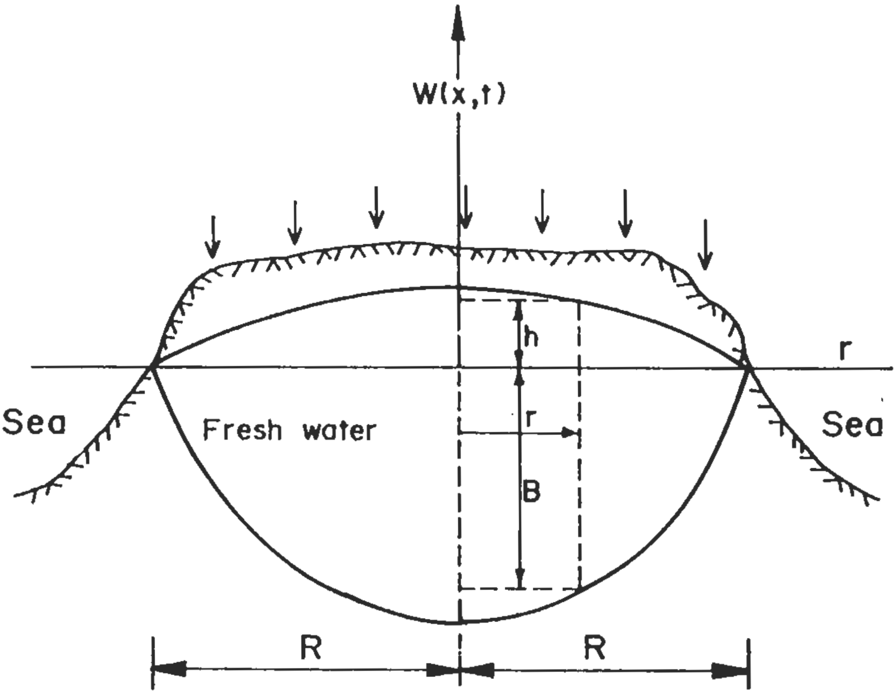

FIG. 6. Axi-symmetrical flow beneath a circular island.

The analytical solutions for these three flows together with the data used are given in Appendix.

Other analytical solutions may be obtained to check the numerical output of the program. However, the satisfactory agreement obtained in these three flow cases indicated that the basic numerical technique and procedure are sound.

Application of Numerical Model to Wadi Hali

1. Description of study area

Wadi Hali, one of the longest wadis in the Kingdom, is located in the Southwestern Region. It starts from Jabal Sawdah in the Abha Region and reaches the Red Sea 60 km to the south of Al Qunfidhah. Figure 8 represents the location of the region. It can be divided into three parts: The upper drainage basin, the middle valley, the lower valley in the coastal plain. The lower valley can be further divided into three main zones, going from east to west: A zone of cultivation, a low zone covered by a very intense vegetation, and the sabkha which is a 3 to 5 km wide coastal strip (SOGREAH 1970).

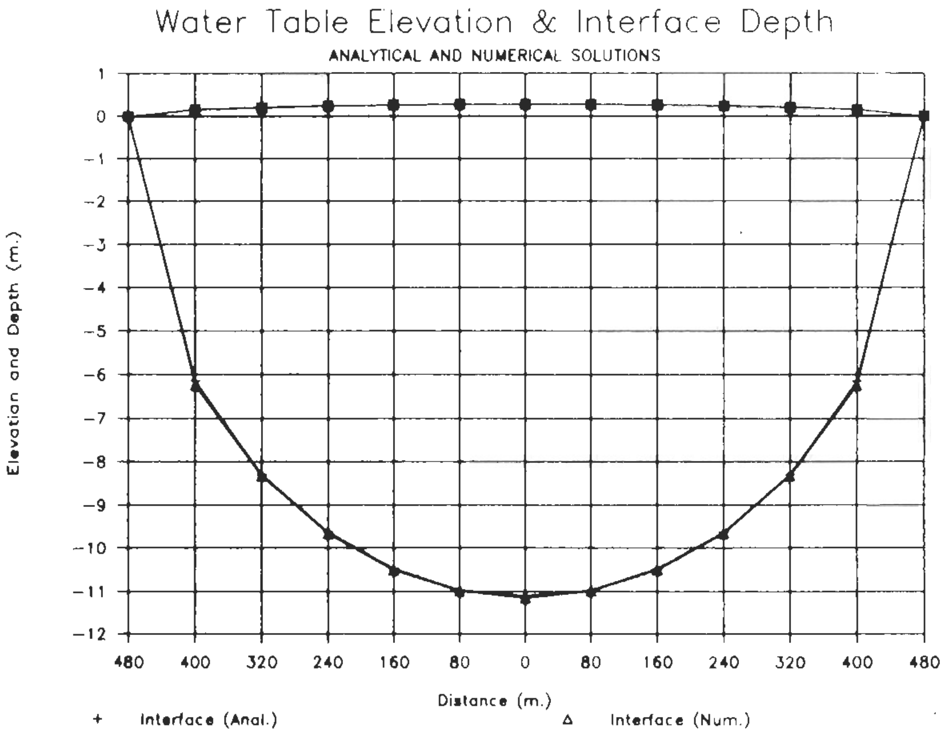

FIG. 7. Variations of water table elevation and interface depth along diameter (Comparison of analytical and numerical solutions).

The significant formations in the coastal plain, from the most recent to the earliest, are as follows: 1. The Alluvia consists of gravel, sand, clayey sand and clay series. Its thickness is variable. 2. The Bayd Formation does not crop out in the coastal plain. It is at shallow depth in the eastern part of the delta and very deep towards the sea. 3. The Basalt Formation forms the Al Birk plateau and stops on the left bank of Wadi Hali on the coastal plain.

Boundaries of the aquifer

The boundaries of the Wadi Hali aquifer on the coastal plain are same as those of the delta. It is limited in the south, by the northern limits of the basalt outflows of the Al Birk Plateau, in the north, by a vast sandy plain. The water table aquifer occupies a sector of a circle whose center is in the Khay-Kiyat area and its periphery along the coast. Its radius is of some twenty kilometers and its apex angle is 70 degree. Upstream of this portion is a long alluvial channel (corridor) limited to the main bed of the wadi between the mountains and Kiyat.

Static water levels

The only available data related to the water levels are based on a field survey made during May 1967, January 1968, July 1968, January 1969 and April 1969. This survey

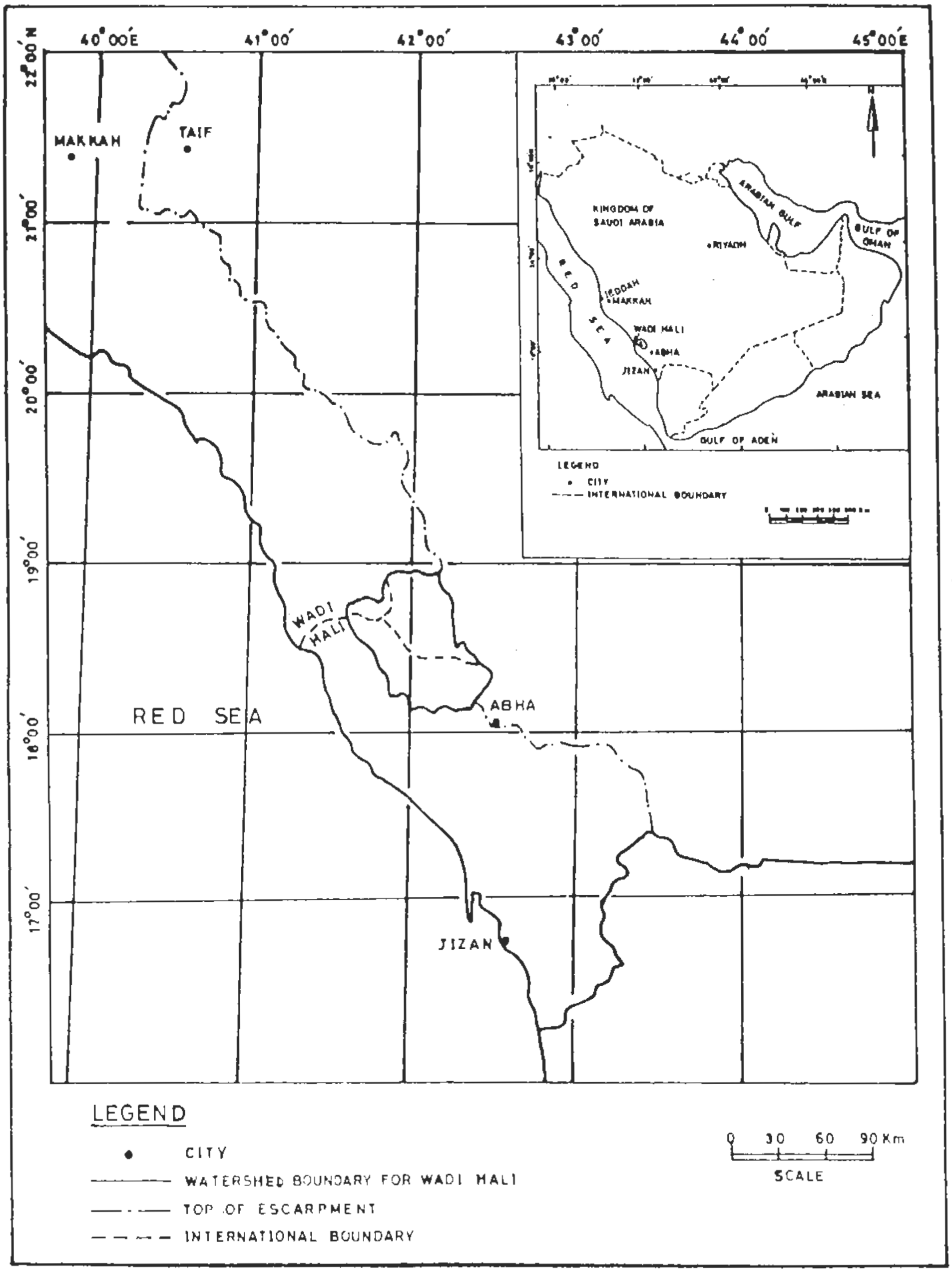

FIG. 8. Location map of Wadi Hali coastal plain.

of over two years indicated that the variation in the water surface elevation is usually in the order of decimeters, except in one or two wells. In other words, over these two years the piezometric surface of the Wadi Hali water table remained almost invariable. Fig. 9 presents static water level contour map of the aquifer (SOGREAH 1970, Fig. 2306-4). This figure has been used in the calibration of the steady-state numerical model.

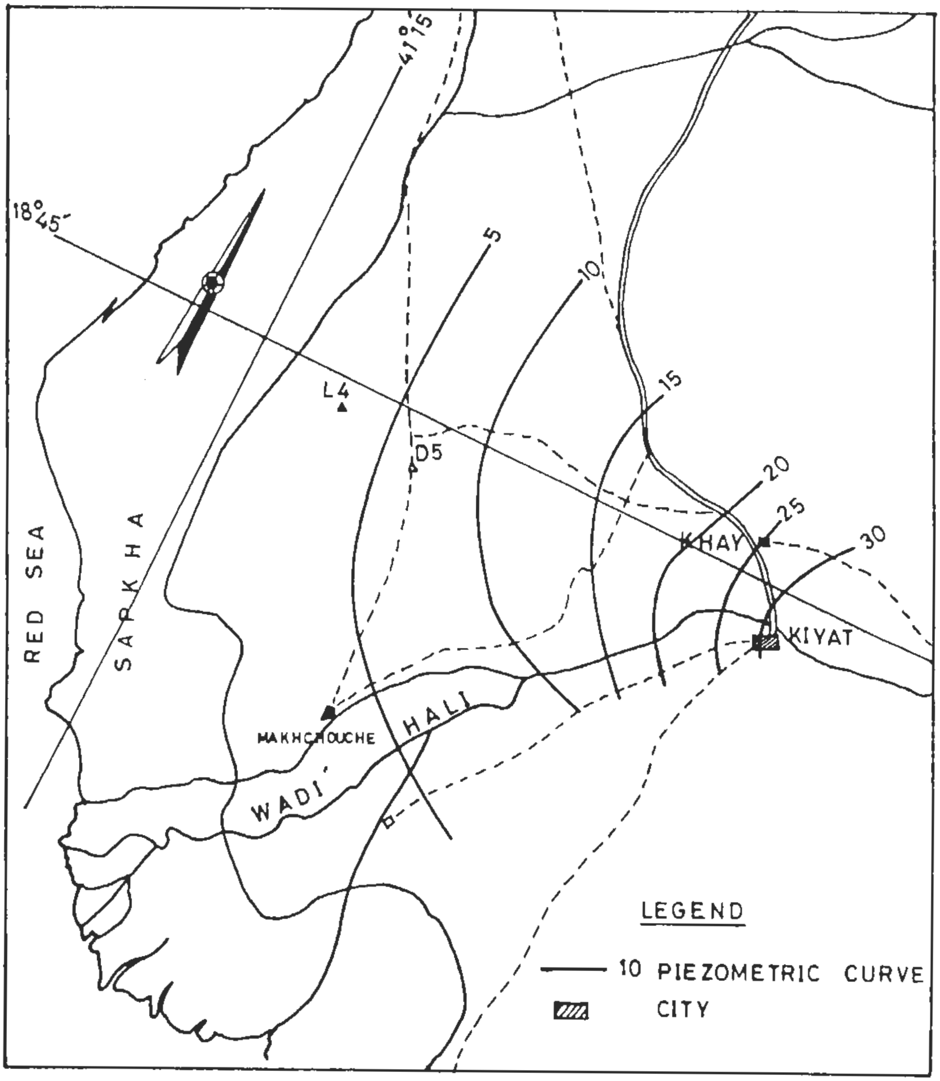

FIG. 9. Piezometric surface map of Wadi Hali.

Hydraulic properties of the aquifer

Results of pumping tests carried out by SOGREAH (1970) in the Wadi Hali delta are summarized in Table 1.

TABLE 1. Hydraulic properties of Wadi Hali coastal plain.

Transmissivity (m^2/s)	Hydraulic conductivity (m/s)	Storage coefficient
1×10^{-2} 3×10^{-1}	4×10^{-4} 1×10^{-3}	4×10^{-3} 9×10^{-3}

The results obtained from field investigations indicate that the aquifer is heterogeneous. Consequently, the pumping test results are only valid for the area immediately surrounding the pumping well. For this reason average values are used in the calibration.

The basic source of the replenishment of the coastal aquifer is the inflow from the narrow gorge and vertical recharge. This amounts to 25×10^6 m^3 per year (SOGREAH 1970). In other words the aquifer is recharged almost exclusively by the seepage from Wadi Hali.

Surface water reaches the sea, only during exceptionally large floods and with no benefits to the aquifer. During the low flow the stream does not reach the sea and all the water seeps down into the ground in the middle valley before it reaches Kiyat.

To the west of the line joining Makhchouche and Qunfidhah the intrusion of the seawater is felt. This can be seen from the geophysical measurements. The electric sounding at a point (D5) located on the line mentioned above showed that the thickness of the fresh water layer is 150 m. This is confirmed by a test borehold (FR 56) made at the same location. The electric sounding at point L4 about 4 km further west indicated that the thickness of the freshwater layer is 50 m.

2. *Model application*

Wadi Hali coastal aquifer is discretized by superimposing a finite difference grid over the map of the aquifer system (Fig. 10). The actual boundary of Wadi Hali was closely approximated. Figure 11 shows the numbering of nodal points.

The input data consisted of the following: Number of columns and rows in finite difference grid; Uniform spacing of grid points; Boundary nodes and boundary conditions; Well locations and rate of pumping in the wells; Rates of recharge and evaporation; Average hydraulic conductivity; Specific weight of fresh and salt water.

Sensitivity analysis

Prior to calibration of the model, in order to determine how variations in hydraulic conductivity (K), net recharge rate (W), and density of the salt water (ρ_s) affect the elevation of water table and the depth of interface, sensitivity analysis are carried

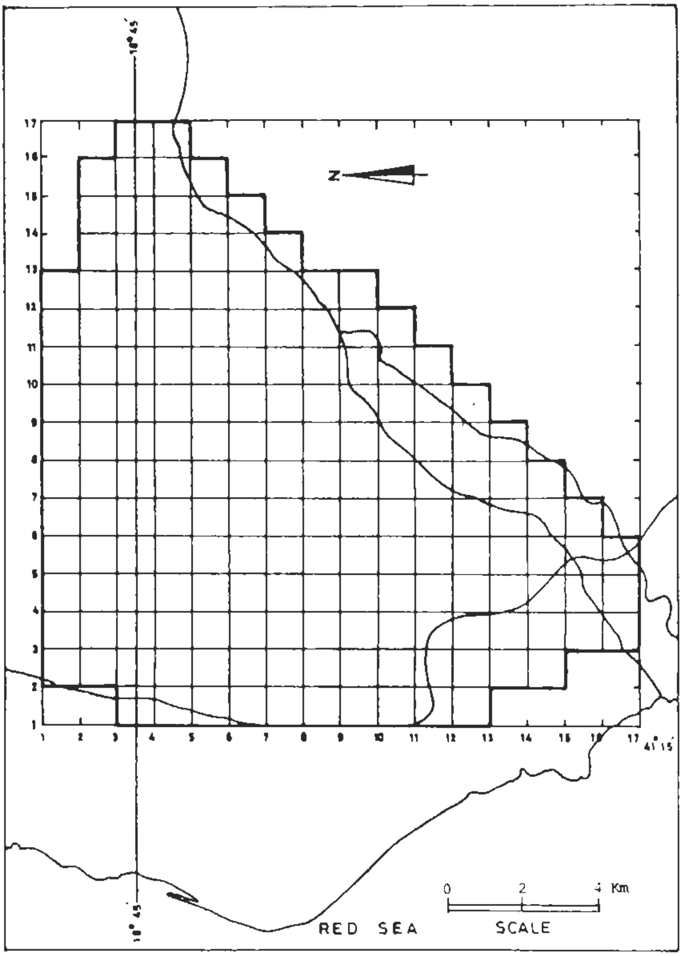

FIG. 10. Finite difference grid over map of aquifer system.

out. It is thought that these are the most influential parameters affecting the aquifer behavior.

The range of variability of the above parameters encompasses most possible values for the chosen aquifer. Table 2 gives the values of the parameters mentioned above.

Results of these analysis indicate the following: a) The sensitivity increases, as the value of hydraulic conductivity decreases. Furthermore, when the hydraulic conductivity is low groundwater mound occurs in the aquifer. b) The variations in the values of net recharge rate produces proportional variations in the water table elevations and in the interface depths. c) The effect of errors in the salt water density will not be

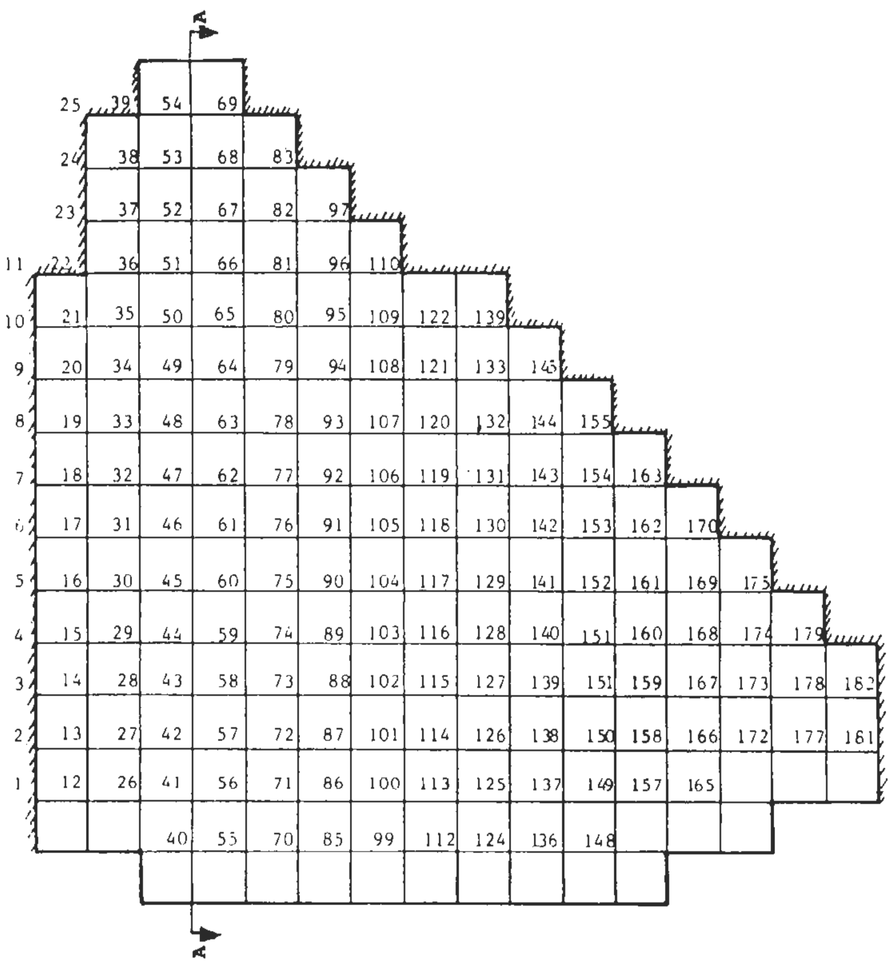

FIG. 11. Numbering of unknown nodal points.

very significant on the aquifer behavior, *i.e.*, the water table elevation and the interface depth.

Figure 12 provides an example which demonstrates the results of sensitivity analysis for recharge rates.

Calibration

Given the input data discussed previously, it is possible to generate a solution for the heads at each nodal point. However, in order to verify the accuracy of the solution, it is necessary to match the computed heads and interface depths with those measured at a number of points in the field. Invariably, the computed values from the first runs of the model will not match the field values. Calibration consists of adjusting the input data until computed values of head and interface depth match the field values. In other words, calibration means that, given a certain combination of

TABLE 2. Values of parameters used in sensitivity analysis.

Run no.	K (m/d)	W (m/d)	Density (kg/m^3)
RSA01	8.64	5×10^{-3}	1030
RSA02	43.2	5×10^{-3}	1030
RSA03	86.4	5×10^{-3}	1030
RSA04	43.2	1×10^{-3}	1030
RSA05	43.2	5×10^{-3}	1030
RSA06	43.2	1×10^{-2}	1030
RSA07	43.2	5×10^{-3}	1025
RSA08	43.2	5×10^{-3}	1029
RSA09	43.2	5×10^{-3}	1033

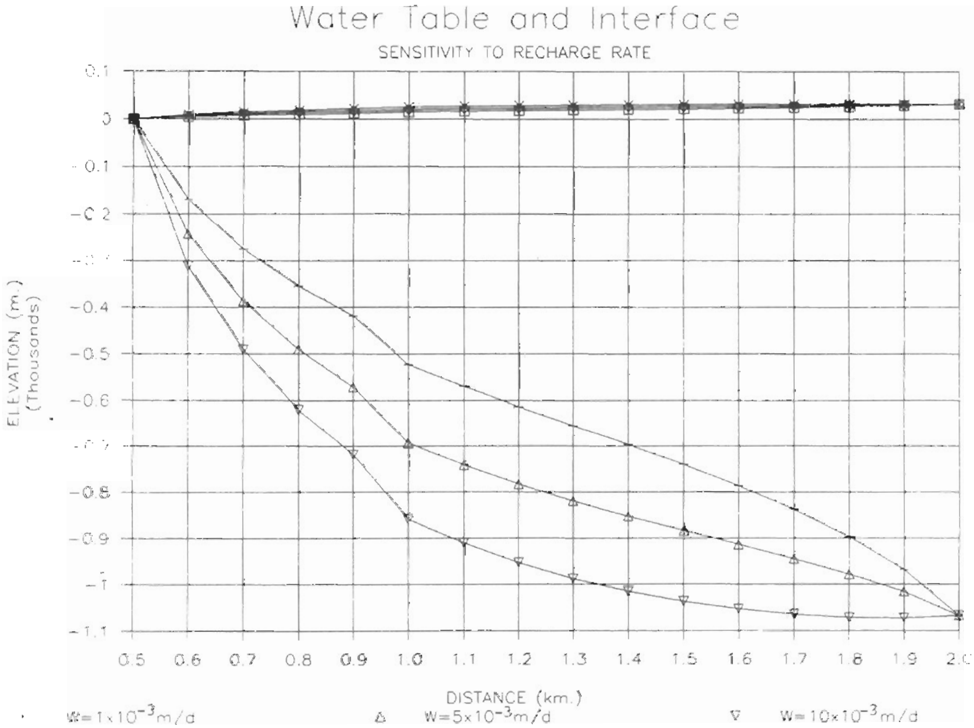

FIG. 12. Sensitivity of water table and interface to recharge rate (Section A-A of Fig. 11).

parameters and boundary conditions, the model will produce field measured values of head at certain points in the grid. However, we have no guarantee that the combination of parameters found by trial and error is unique (Wang and Anderson 1982). It is not unreasonable to adjust the input data because these data are imperfectfly known, and there will be a certain range of values that may be valid. During calibration process, around forty trial and error simulations have been made, and the results of sensitivity analysis have been fruitful in reaching the final adjustment. The results of calibration are shown in Fig. 13 and 14.

Field data used in Fig. 13 and 14 are taken from SOGREAH (1970). The determination of the interface depth has been done through electric logging carried out in boreholes. Also, sampling in wells and boreholes was done and electric conductivity

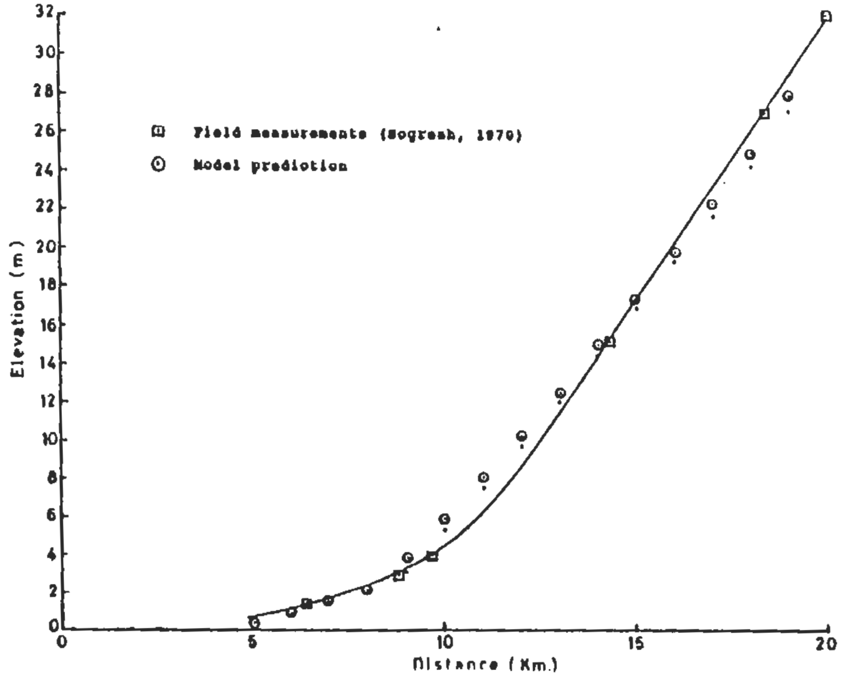

FIG. 13. Comparison of model prediction with observed water table (Section A-A of Fig. 11).

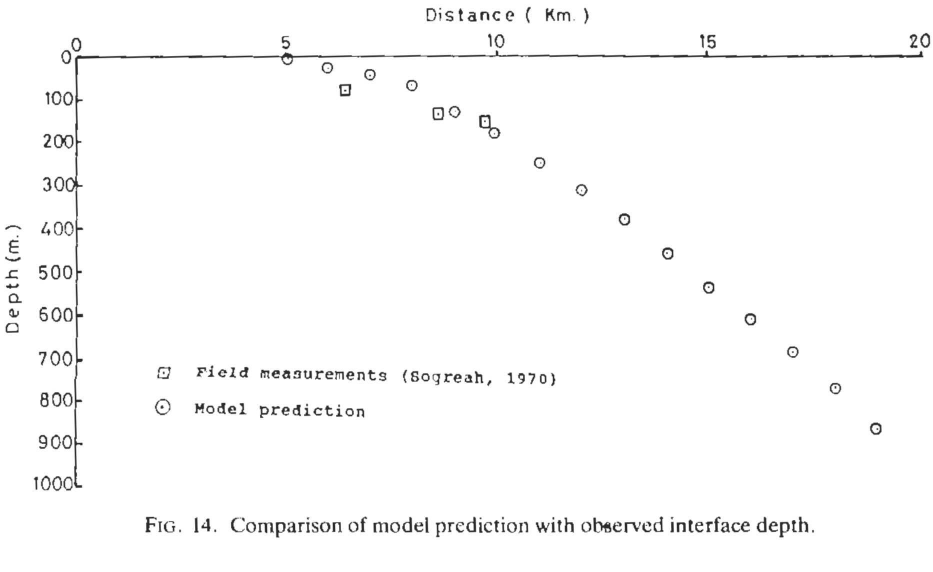

FIG. 14. Comparison of model prediction with observed interface depth.

measurements were carried out to determine the salinity.

Results of simulation

The main goal of the groundwater hydrologist, water resources engineers, or planner, who deals with a groundwater system, is the management of the groundwater system. In this context the purpose of computer simulation is to predict the effect of some proposed management scheme on the groundwater system.

To demonstrate the applicability of the model as a tool in the management activities, the following scenarios are considered as examples.

Let us assume that a series of wells are planned to be placed in the cultivation zone on a line extending from south to north. Suppose that this is a requirement of a possible future agricultural expansion plan. Let Q, the discharge per well is 99 m^3/day and the number of wells is 17. The groundwater resources manager, or planner would like to know what the new configuration of the water table and the interface depth would be? To answer this question, the calibrated model is run with this information as input. The results indicated clearly the difference between present situation and the one after the development and a decline of up to 0.25 m in the water table is predicted by the model (Fig. 15). This decline is zero in Kiyat area and maximum toward the sea. The present and future depths of the interface are also calculated by the model. Figure 16 indicates that the salt water interface rises up to 8 m after development.

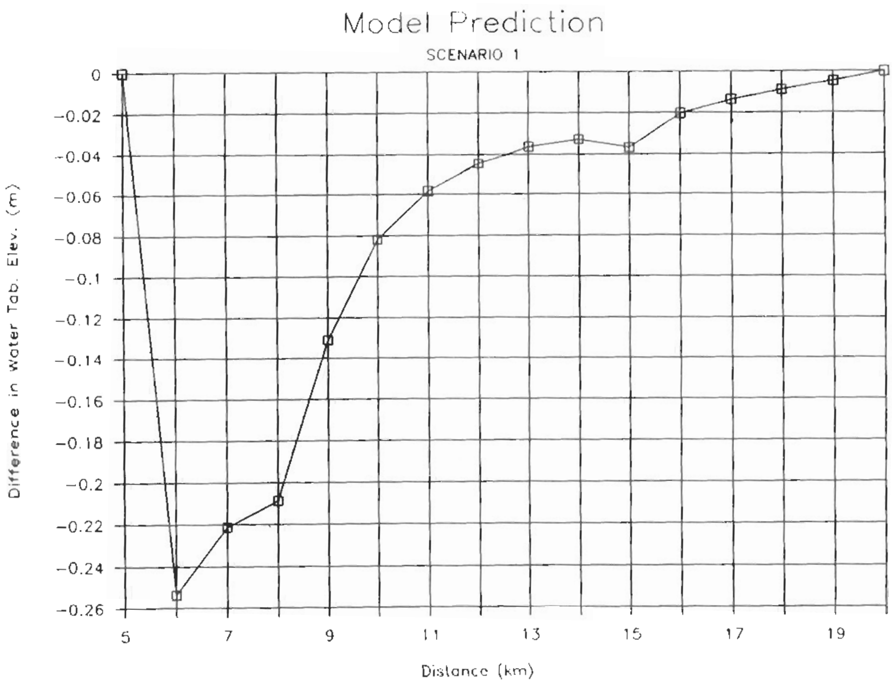

FIG. 15. Decline in water table elevation (Scenario 1).

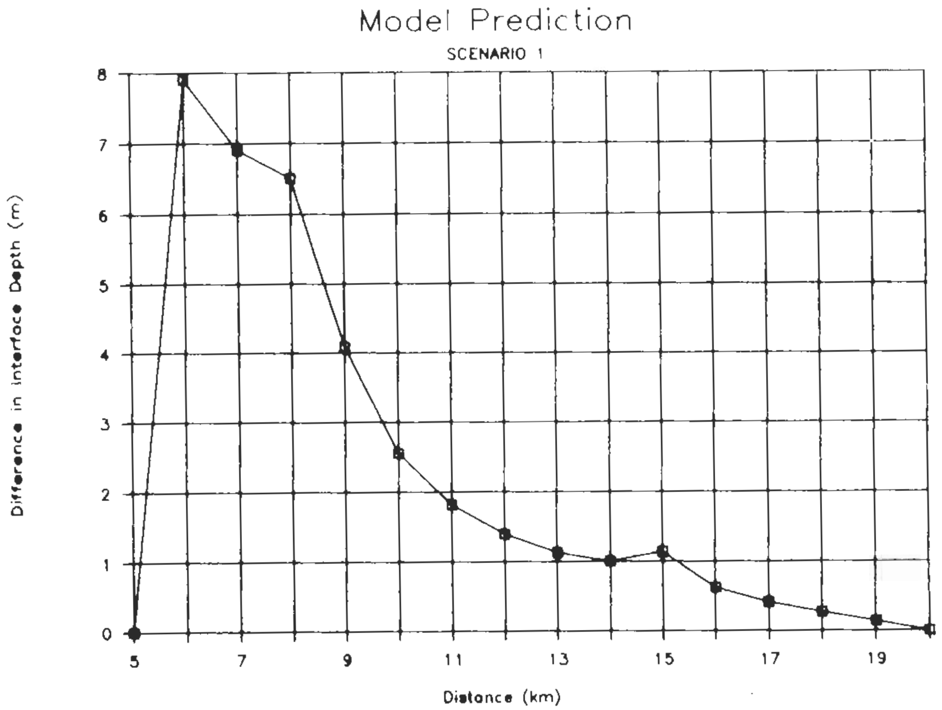

FIG. 16. Rise in interface depth (Scenario 1).

Let us consider a slightly different scenario, in which the wells are located more upstream toward Kiyat in the eastern part of cultivation zone. Let the number of wells be only three (at nodes 35, 65, and 95 of Fig. 11). Assume that the wells are pumping a total amount of 6003 m^3/day. Assume also that a total amount of 1717 m^3/day is recharged back into aquifer from excess agriculture irrigation water. The results for this case showed that maximum lowering in the water table elevation is 0.75 m (Fig. 17), while maximum rise in the interface depth is approximately 24 m (Fig. 18). A more important point to be noted is that salt water wedge advanced approximately one km more toward inland.

Finally one more case is considered. It is hypothetically assumed that there is no underflow from Kiyat area toward the coastal plain. Since the underflow is the only source of replenishment for the aquifer, in the long run, the aquifer would be invaded completely with salt water. In effect the results of the model confirmed this by producing zero water table elevations and zero interface depths.

Discussion and Conclusion

Numerical simulation of groundwater flow systems depends, to a large extent, upon the availability of field data. The data related to the salt water interface prob-

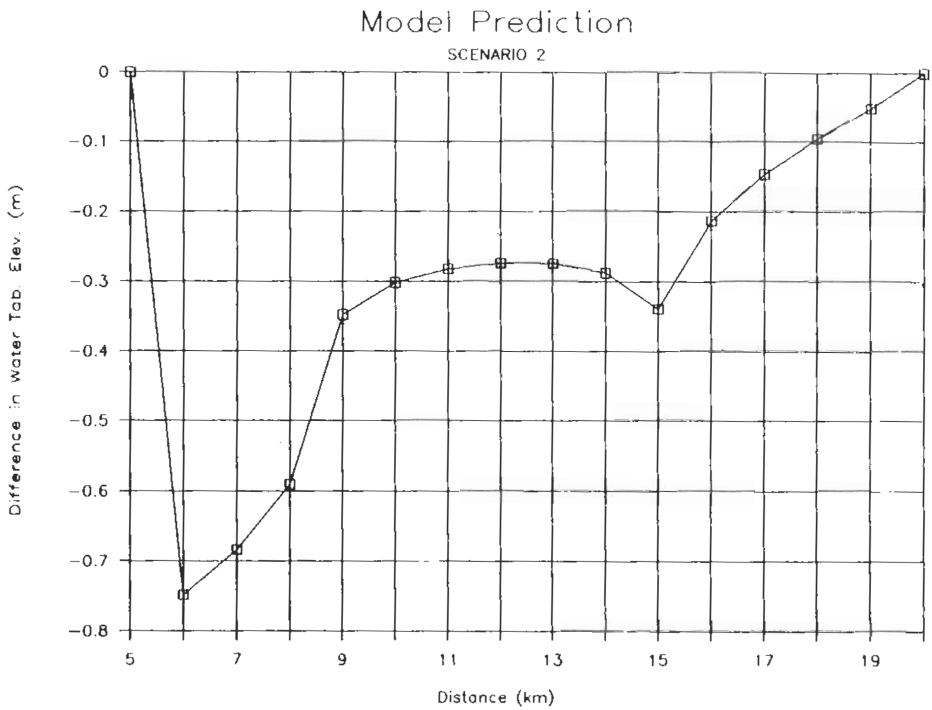

FIG. 17. Decline in water table elevation (Scenario 2).

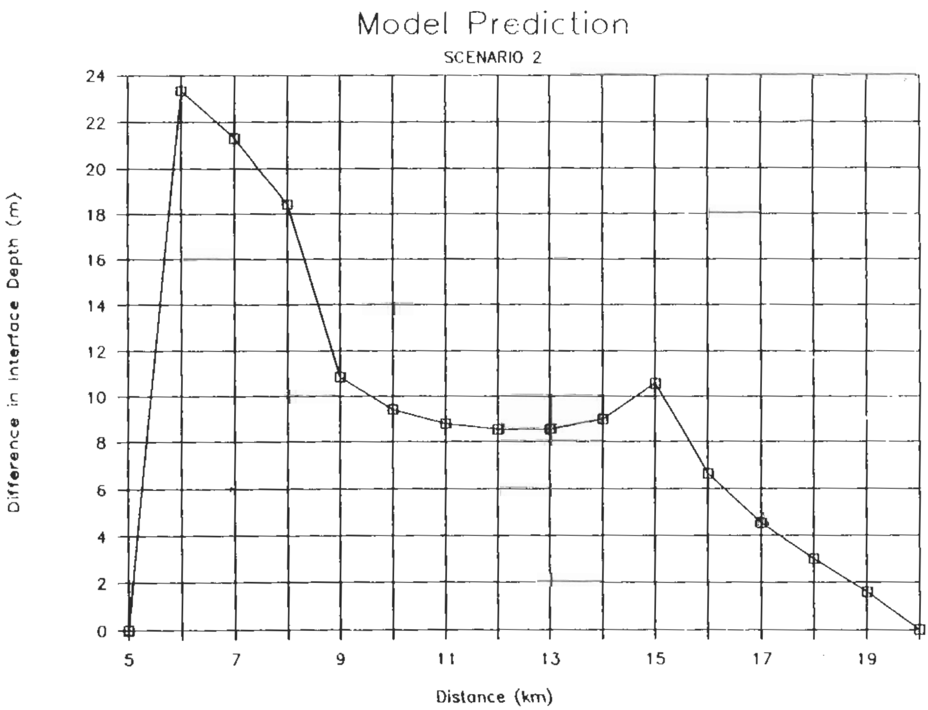

FIG. 18. Rise in interface depth (Scenario 2).

lems in available technical report is, however, very scarce. This is one of the major difficulties that the authors have faced.

The inaccuracies in aquifer hydraulic properties and boundary conditions together with the difficulties and uncertainties in the determination of the exact subsurface geometry of the aquifer introduce some errors in the simulation results. On the other hand, since Dupuit-Forchheimer assumptions and sharp interface assumption of Ghyben-Herzberg are used in the present numerical model, additional errors are also introduced. However, it is believed that the former overshadows the latter.

The two-dimensional numerical model, in spite of its inherent restrictions, is a useful tool for the prediction of the water table elevation and the interface depth. When steady-state local conditions are of interest, it can be successfully used in the management of the groundwater resources (Bear *et al.* 1985). When the flow is time dependent, it can be approximated by successive steady states, and the proposed model can still be used for simulation purposes. Particularly, when the model is suitably linearized, it can be very helpful for incorporation into a management model of an aquifer in which the location and the discharge rates of the pumping wells are decision variables.

The depth to salt water interface is sensitive to hydraulic conductivity, therefore the determination of field hydraulic conductivity values should be given due attention.

The model takes into account the spatial variability of the recharge and evaporation rates. The rates of evaporation and recharge are controlled by the vegetation cover and surface soil characteristics. This aspect is important when these parameters vary spatially.

Careful placement of the recharge or extraction wells and proper selection of their rates is essential for the management of groundwater resources.

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List of Symbols

A Coefficient matrix.
$a_{i,j}$ Elements of coefficient matrix A.
B Column matrix.
b_i Elements of column matrix B.
h Elevation of water table above mean sea level.
K Hydraulic conductivity.
L Width of the strip aquifer.
Q Discharge.
S Specific yield.
t Time.
V Column matrix of unknowns.
$v_{i,j}$ Elements of column matrix V ($v = h^2$).
W Net recharge rate (accretion).
x Space coordinate in x-direction.
y Space coordinate in y-direction.
Δx Space increment in x-direction.
$\delta = \gamma_f / (\gamma_s - \gamma_f)$
γ_f Specific weight of freshwater.
γ_s Specific weight of salt water.
η Depth of salt water-freshwater interface below mean sea level.
ρ_f Density of freshwater.
ρ_s Density of salt water.

Appendix

Analytical Solutions

The analytical solutions for the flow cases described in third section of the text (Verification of Numerical Model with Analytical Solutions) are provided by the following equations. In these equations the variation of the water table elevation, h, with respect to mean sea level is given as a function of the linear distance, x, or the radial distance, r. The symbols are defined in the List of Symbols. For details see Onder (1989), and Onder and Hassan (1989).

Strip aquifer

a) Strip aquifer between sea and a freshwater lake:

$$h = h_o^2 - \frac{W}{K(1+\delta)} x^2 - \left[\frac{h^2}{L} - \frac{WL}{K(1+\delta)} \right] x \qquad \text{(A - 1)}$$

b) Strip aquifer between sea and an impervious boundary:

$$h = \frac{W}{K(1+\delta)}(L^2 - x^2) \tag{A - 2}$$

Circular aquifer

$$h = \frac{W}{2K(1+\delta)}(r^2 - R^2) \tag{A - 3}$$

Data

Strip aquifer

The following data is considered for both cases: For a case given above the water level elevation in the lake is taken as 1.5 m.

Width of the strip aquifer	400 m
Hydraulic conductivity	1 m/day
Net recharge rate	0.0001 m/day
Density of the freshwater	1000 kg/m^3
Density of the salt water	1025 kg/m^3
Mash size	100 m by 100 m

Circular aquifer

The following data is considered.

Diameter of the aquifer	960 m
Hydraulic conductivity	20 m/day
Net recharge rate	0.0002 m/day
Density of the freshwater	1000 kg/m^3
Density of the salt water	1025 kg/m^3
Mash size	80 m hy 80 m

تحليل تداخلات المياه الملحة مع المياه العذبة في الخزانات الجوفية الساحلية

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المستخلص . ركزت هذه الدراسة على موضوع تداخل المياه الملحة مع المياه العذبة في المناطق الساحلية ، حيث تم صياغة المعادلات التفاضلية الجزئية المتحكمة في هذا التداخل ، وذلك بربط معادلة حركة المياه لدارسي بمعادلة الاستمرارية بمبدأ غيبن هيرزبيرج ، وفرضيات ديبويت . استخدمت طريقة الفروق العددية المتناهية باستخدام برنامج كمبيوتر ذو بعدين لحل هذه المعادلات .

ونظرًا لعدم توافر البيانات الحقلية الخاصة لإثبات نتائج الدراسة ، فقد تم استخدام برنامج الكمبيوتر لتطبيقه على حالتين مبسطتين ، مع توافر الحل التحليلي ، وهما :

الأولى : حركة المياه في اتجاه واحد لشرطين حدوديين مختلفين .

الثانية : حركة المياه الدائرية في اتجاه الخزان الأرضي .

وبتطبيق النموذج ، أسفرت النتائج عن وجود توافق جيد بين نتائج الطريقتين التحليلية والعددية .

وبعد معايرة النموذج واختبار حساسيته ، تم تطبيقه على المناطق السفلى من وادي حلى في المنطقة الساحلية الغربة من المملكة العربية السعودية ، ويعد هذا التطبيق كمثال على التخطيط الإداري ، وتحتوي هذه الدراسة على نتائج تطبيق هذا النموذج وكذلك الاستنتاجات المستخلصة من تطبيقه . أما بالنسبة لمحدودية استخدام هذا النموذج ، فهي بسبب وجود بعض الافتراضات ، وهي أن هذا النموذج يطبق فقط في حالة حركة المياه المنتظمة في اتجاهين لتربة متجانسة على طول منطقة الدراسة .